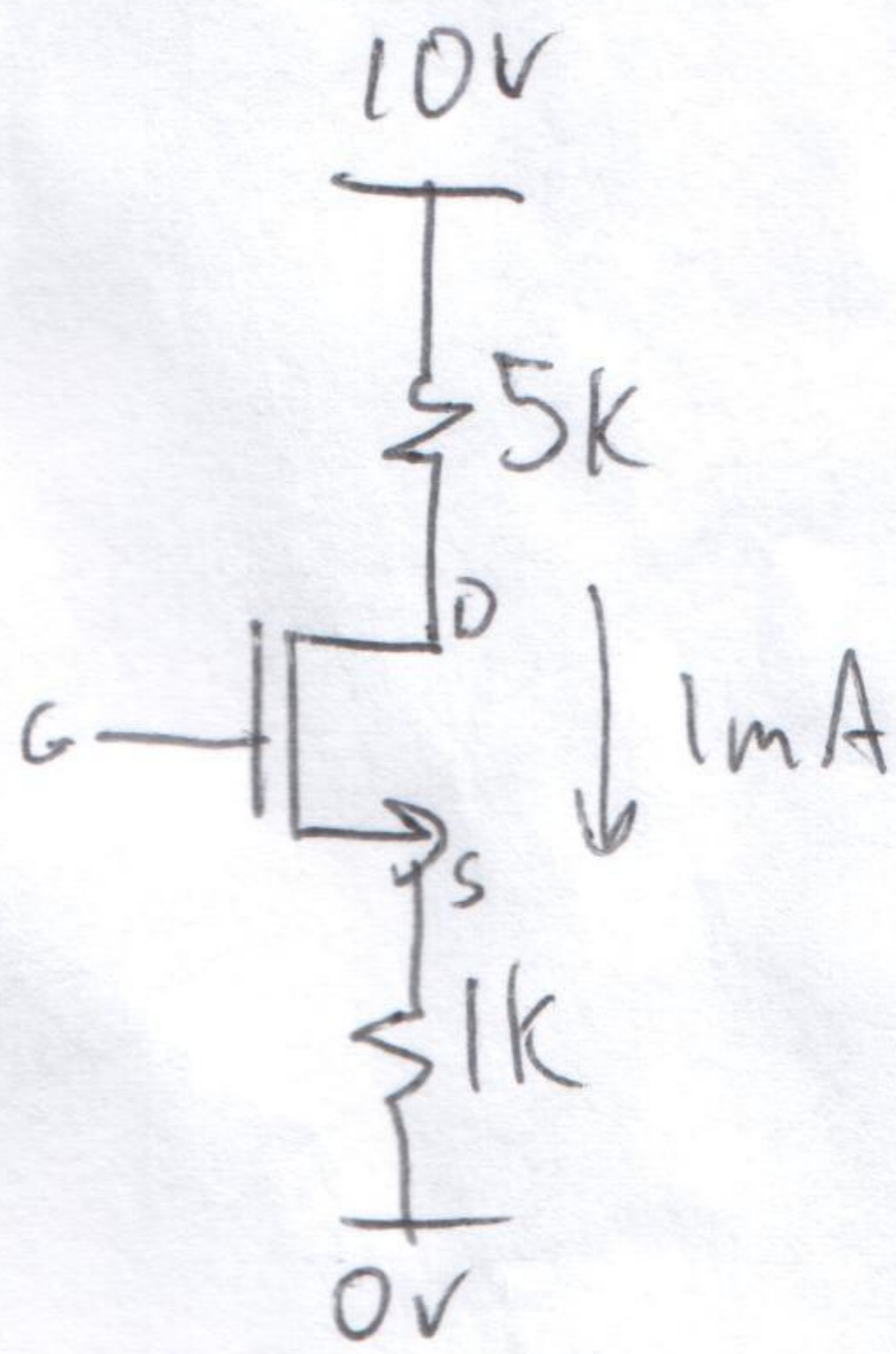


This circuit is very difficult to calculate  $I_D$  from  $V_G$  because spurious quadratic roots are everywhere.

In practice this is almost never calculated without knowing something of the situation in advance.

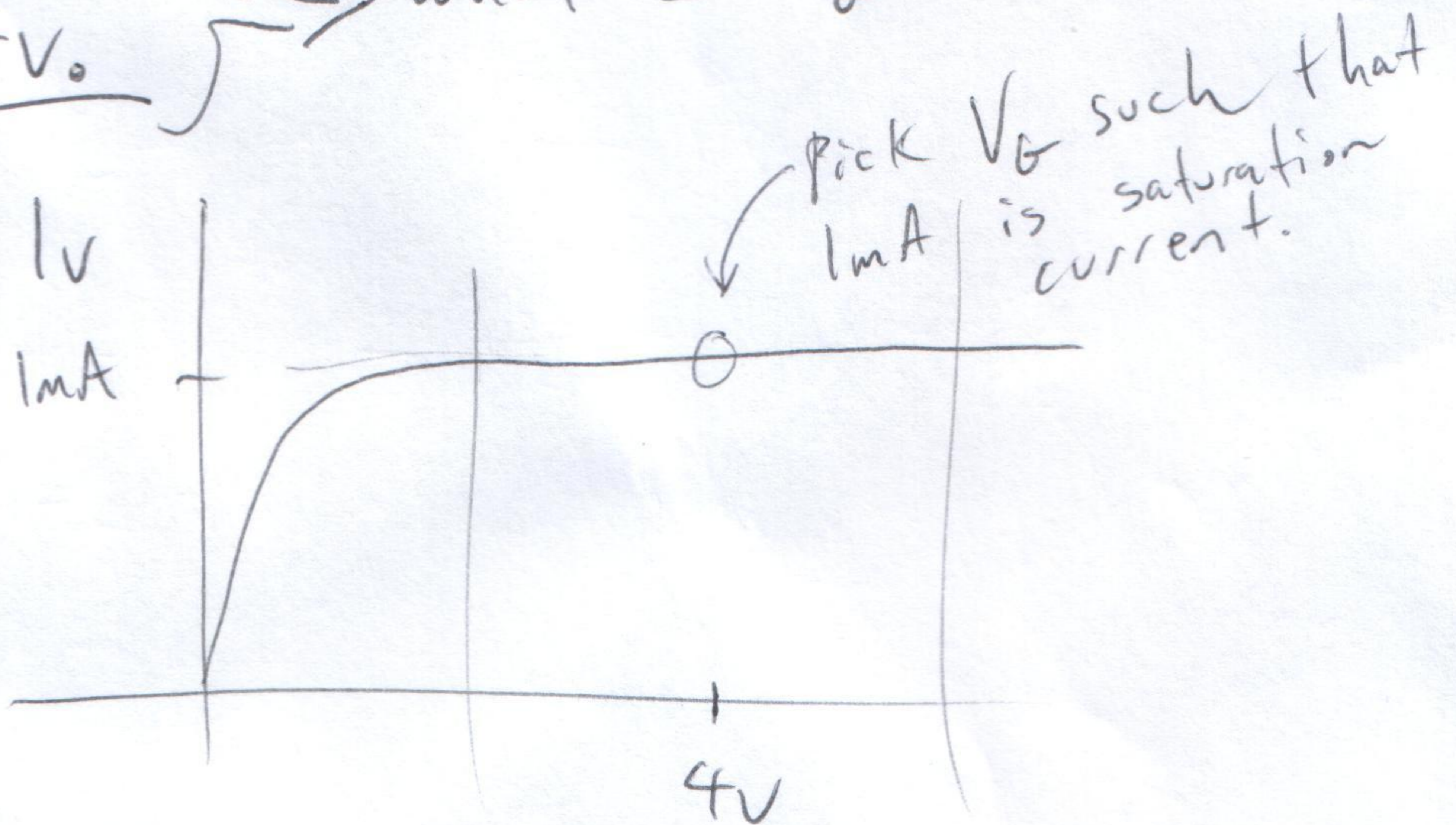
Specifically, real circuit engineers have a plan.



$I_D = 1\text{mA}$   
 $V_{DS} = 4V_0$

what is  $V_G$ ?

$V_S = 1V$   
 $1\text{mA}$



$$I_D = K_n \frac{W}{L} (V_{GS} - V_{TN})^2$$

$$1\text{mA} = K_n \frac{W}{L} (V_{GS} - 1)^2$$

Notice, we can set this with either of these.

In fact: Pick  $V_{GS} = 5V$ .

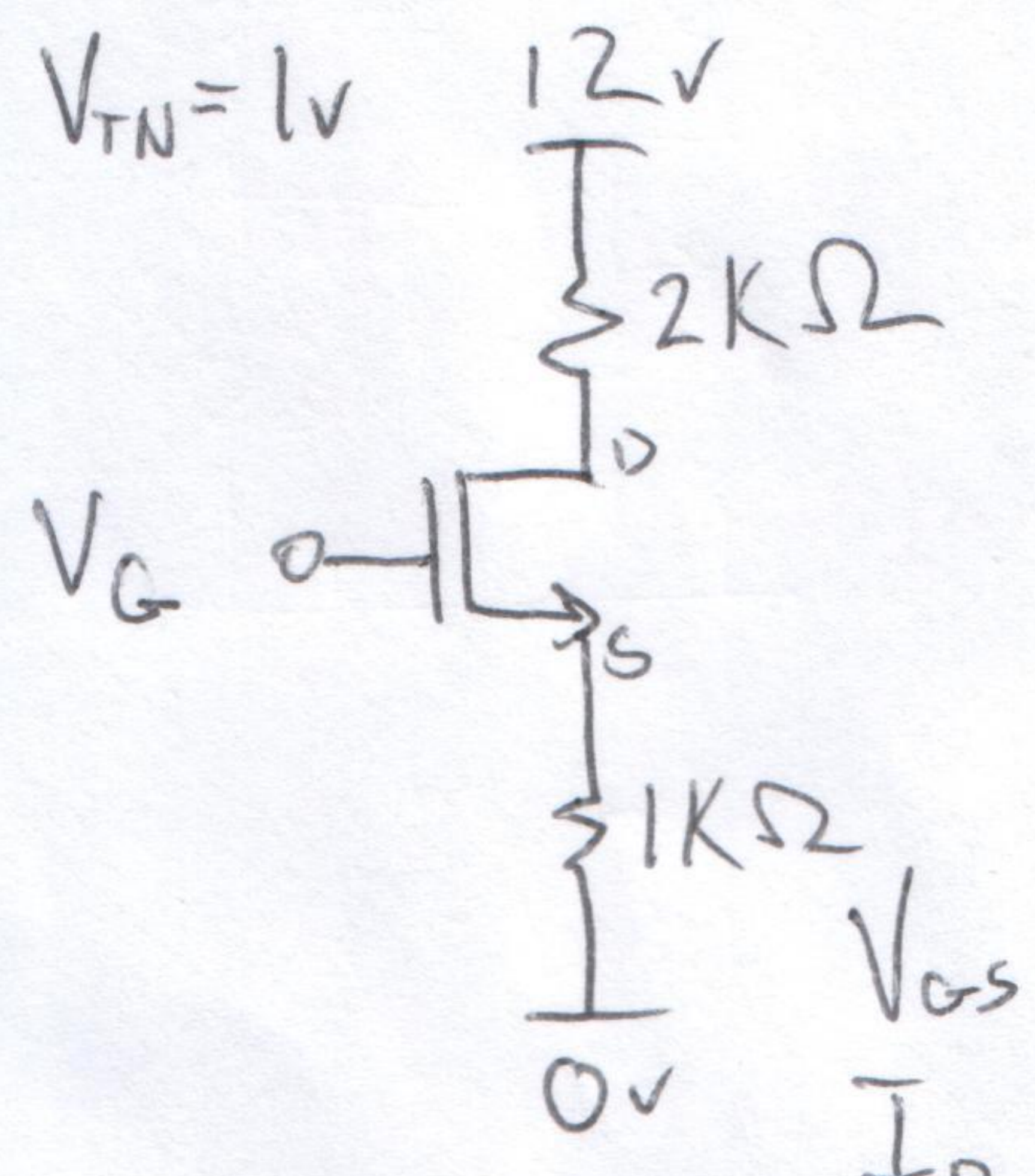
$$K_n \frac{W}{L} = \frac{1}{16} \text{mA}$$

if  $K_n' = 60 \mu\text{A}/\text{V}^2$ ,

$$\frac{W}{L} = 625/1$$

sounds pretty rough, but really happens!

$K_n = 10 \text{ mA/V}^2$   
 $\frac{W}{L} = 10$   
 $V_{TN} = 1 \text{ V}$



$V_G = 5 \text{ V}$ : Solve... But don't get stuck in a forest of spurious quadratic roots! (or at least learn something!)

All values of  $V_{GS}$  have a max current, set by  $V_{GS} @ \text{Sat}$ . Find that for some values of  $V_{GS}$ :

|                  |                  |                     |                   |                   |                   |                   |                   |
|------------------|------------------|---------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1.5              | 2                | 2.5                 | 3                 | 3.5               | 4                 | 4.5               | 5                 |
| 12 $\mu\text{A}$ | 50 $\mu\text{A}$ | 112.5 $\mu\text{A}$ | 200 $\mu\text{A}$ | 312 $\mu\text{A}$ | 450 $\mu\text{A}$ | 612 $\mu\text{A}$ | 800 $\mu\text{A}$ |

$I_{D \text{ SAT}} = 100 \mu\text{A} \frac{1}{2} (V_{GS}^{\text{MAX}} - V_{TN})^2$

(eight values is probably excessive, but can be handy.) (and it's easy with Matlab/Octave/etc)

$V_{DS} = 12 - I_D \cdot 3\text{K}$

|       |       |       |      |      |       |       |     |
|-------|-------|-------|------|------|-------|-------|-----|
| 12.96 | 11.85 | 11.66 | 11.4 | 11.1 | 10.65 | 10.16 | 9.6 |
|-------|-------|-------|------|------|-------|-------|-----|

now let's calculate  $V_{GS}$  by:  $V_G = I_D \cdot 1\text{K} + V_{GS}$

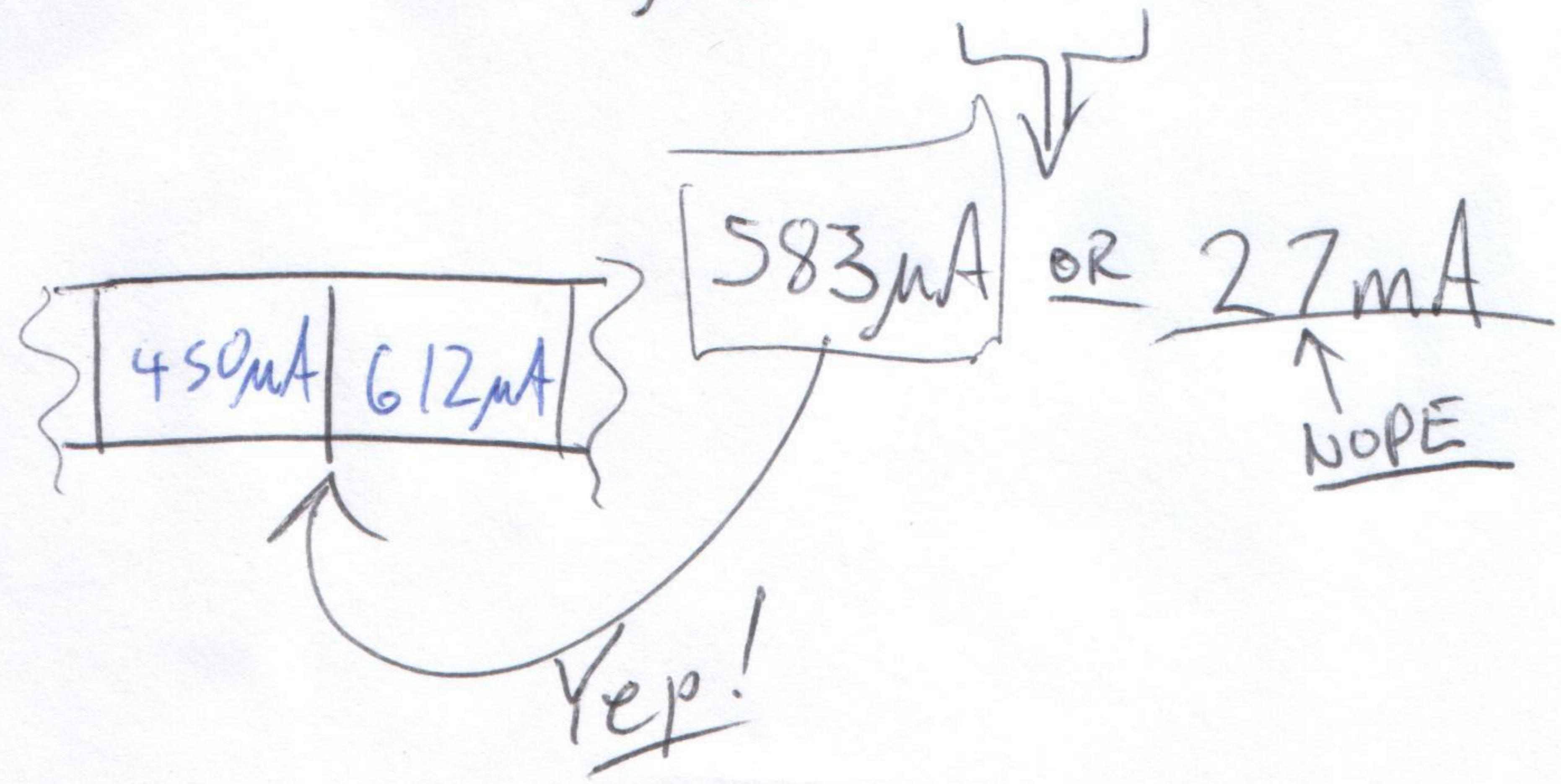
|      |      |      |     |     |      |      |     |
|------|------|------|-----|-----|------|------|-----|
| 1.51 | 2.05 | 2.61 | 3.2 | 3.8 | 4.45 | 5.11 | 5.8 |
|------|------|------|-----|-----|------|------|-----|

Note: Definitely nowhere near linear.

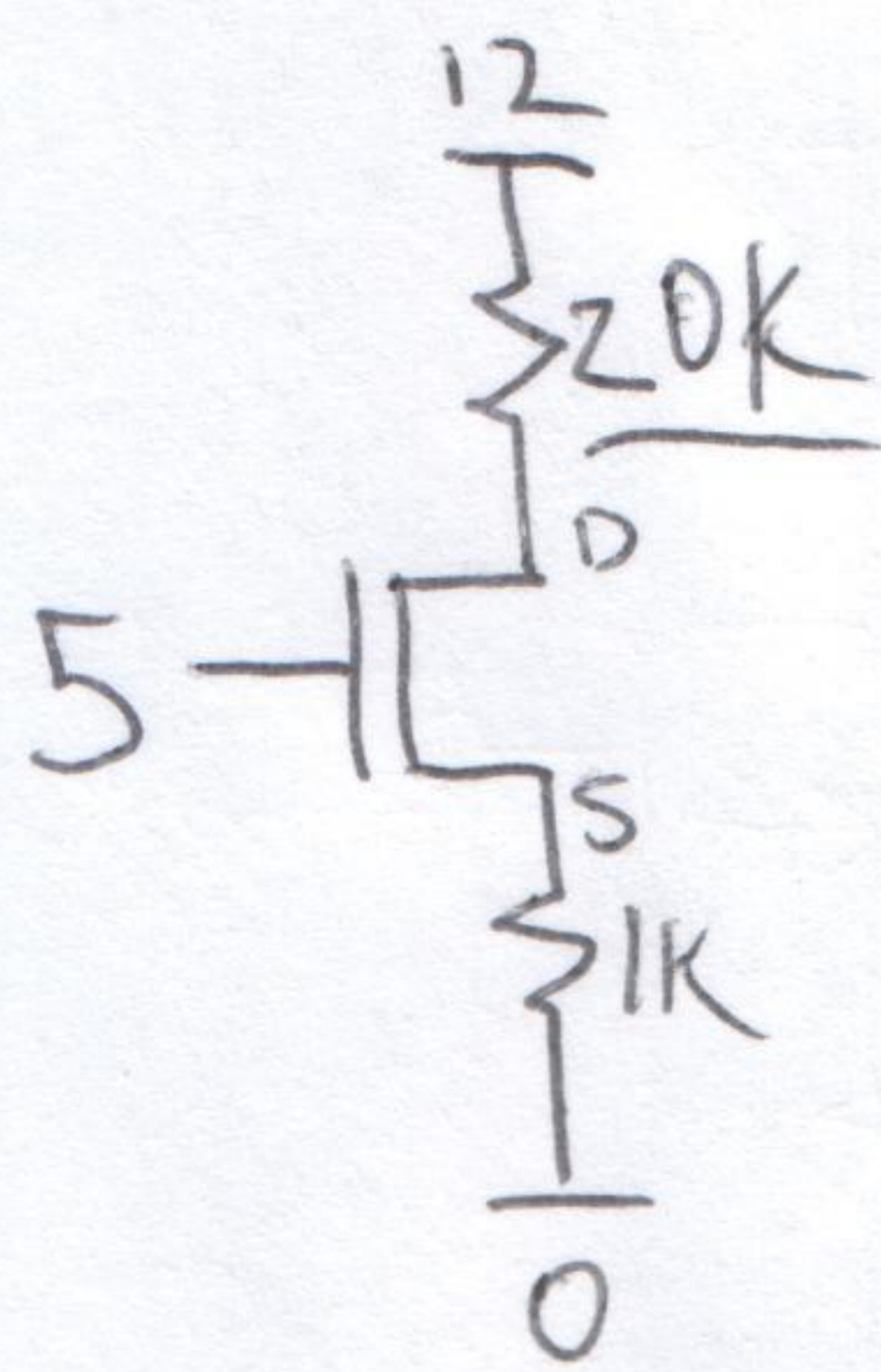
actual value of circuit current probably about there.

**Now** you calculate, with a fairly good grip on what answer you should get.

$I_D = 50 \mu (5 - V_S - V_{TN})^2$        $V_S = 1\text{K} \cdot I_D$



But what if it ended up linear.



|          |            |            |               |             |             |             |             |             |
|----------|------------|------------|---------------|-------------|-------------|-------------|-------------|-------------|
| $V_{GS}$ | 1.5        | 2          | 2.5           | 3           | 3.5         | 4           | 4.5         | 5           |
| $I_D$    | 12 $\mu$ A | 50 $\mu$ A | 112.5 $\mu$ A | 200 $\mu$ A | 312 $\mu$ A | 450 $\mu$ A | 612 $\mu$ A | 800 $\mu$ A |

These don't change because  $V_G$  is the same and so is the trans.

$V_{DS}$  (at sat)  $[V_{DS} = 12 - I_D \cdot 20K]$

|                |      |      |     |     |      |      |       |      |
|----------------|------|------|-----|-----|------|------|-------|------|
| $V_{DS}$ (sat) | 11.7 | 10.9 | 9.6 | 7.8 | 5.43 | 2.55 | -0.86 | -4.8 |
|----------------|------|------|-----|-----|------|------|-------|------|

$V_G = I_D \cdot 1K + V_{GS}$

$V_{GS} < V_{TH}$  Saturation Can't Be Valid Here

|       |      |      |      |     |     |   |   |   |
|-------|------|------|------|-----|-----|---|---|---|
| $V_G$ | 1.51 | 2.05 | 2.61 | 3.2 | 3.8 | X | X | X |
|-------|------|------|------|-----|-----|---|---|---|

NOT AVAILABLE

We've learned from this that if  $V_{GS} > 3.5$ , it's probably linear, and further that  $V_{GS}$  almost certainly is.

So: Linear:

$$I_D = 100\mu \left( 5 - V_S - V_{TN} - \frac{V_D - V_S}{2} \right) (V_D - V_S)$$

$$V_D = 12 - 20KI_D \quad V_S = I_D \cdot 1K$$

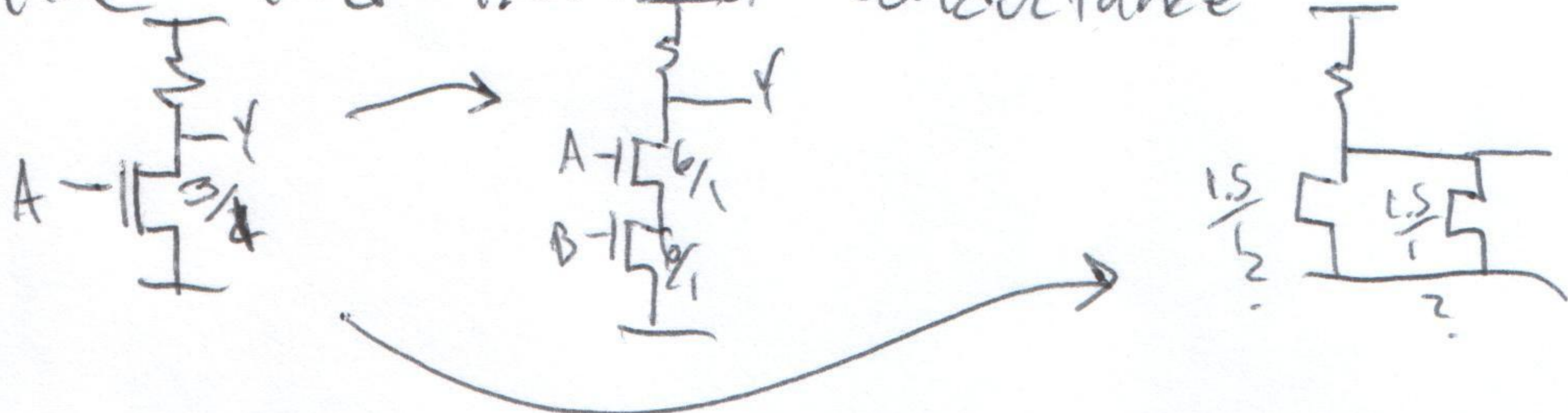
$$I_D = 100\mu \left( 4 - I_D \cdot 1K - \frac{12 - 21KI_D}{2} \right) (12 - 21KI_D)$$

$$I_D = 100\mu (-2 + 10.5KI_D) (12 - 21KI_D)$$

$218\mu$ A OR  $498\mu$ A

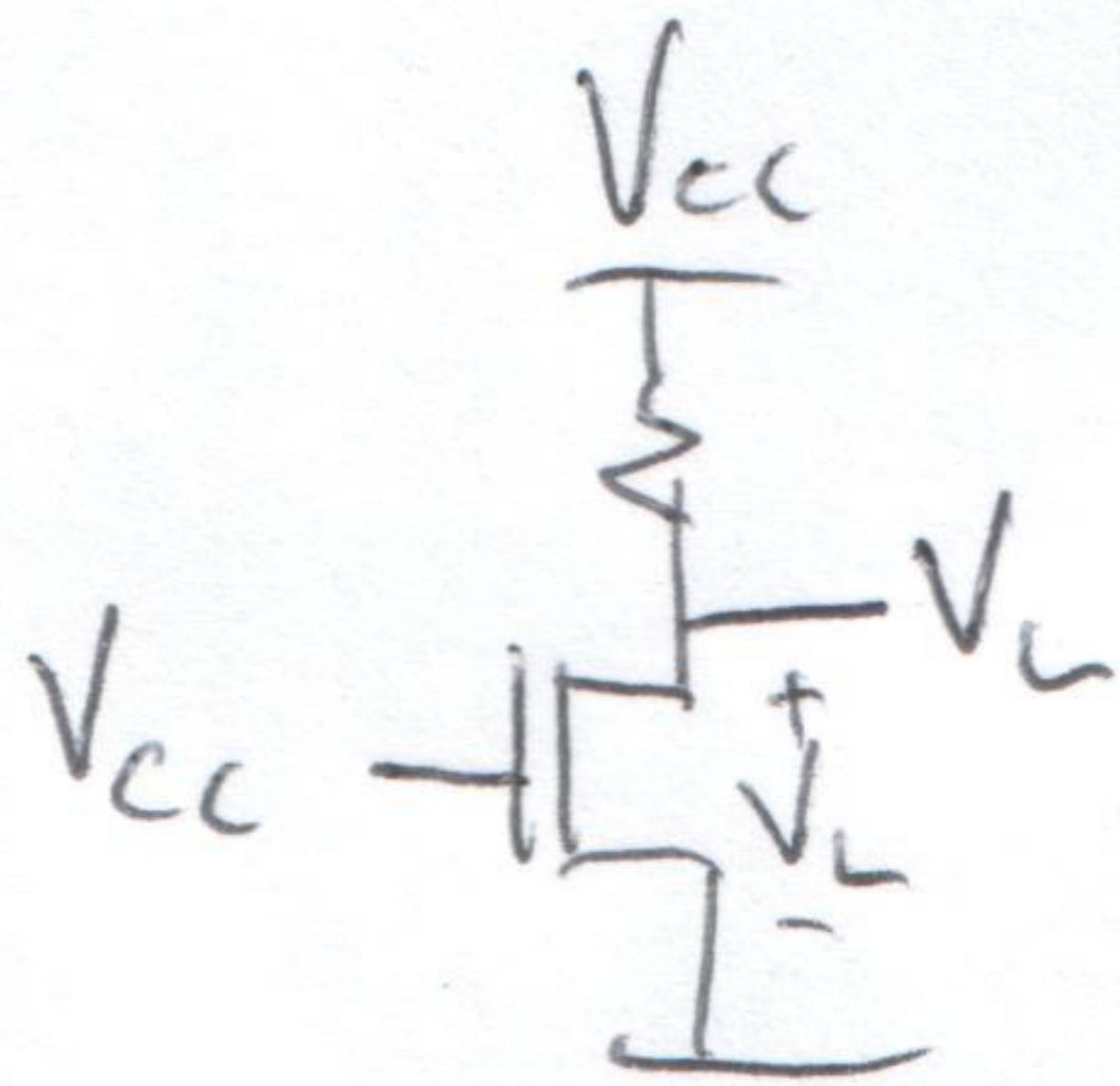
Both potentially valid. Iterate between  $V_G$  &  $I_D$  to be sure.

# Resistance and Transistor "conductance"



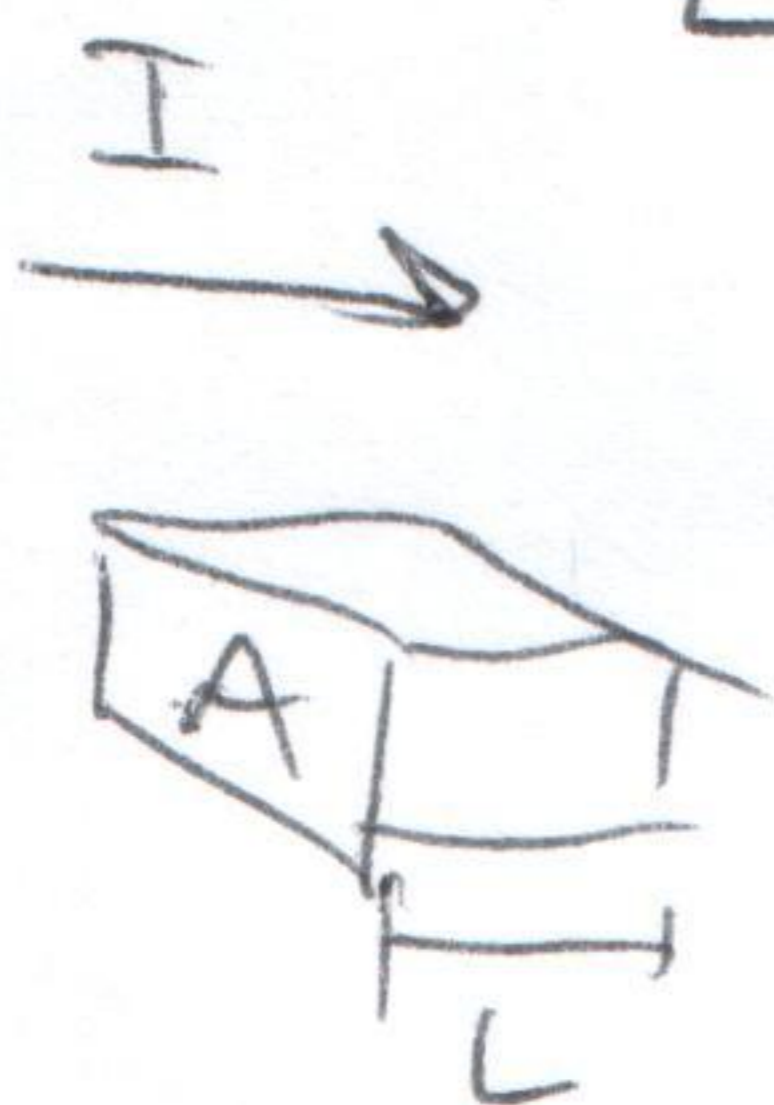
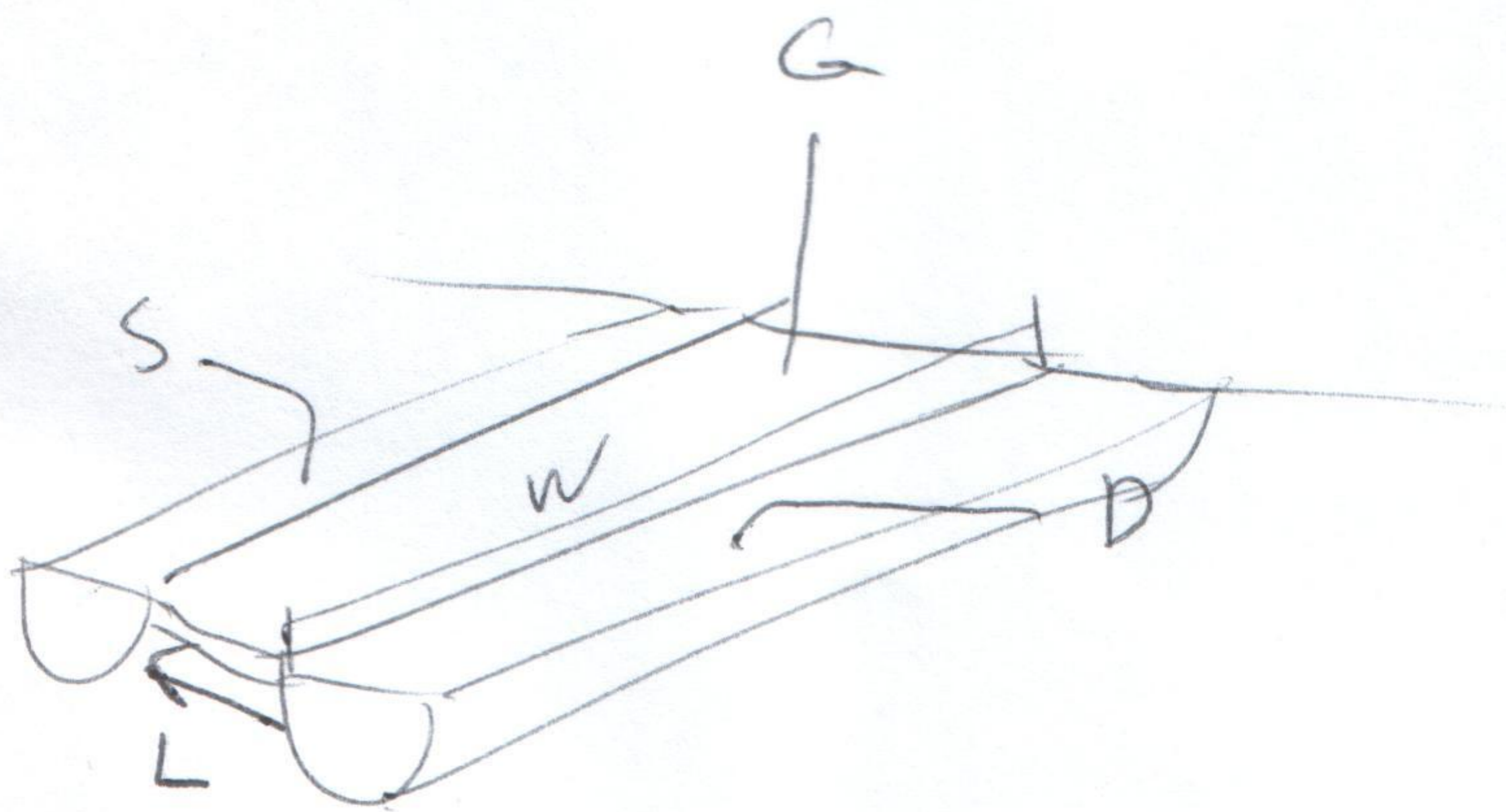
$$I_D = \frac{V_{CC} - V_L}{R} = k_n' \frac{W}{L} \left( V_{GS} - V_{TN} - \frac{V_L}{2} \right) V_L$$

Solve this monstrosity

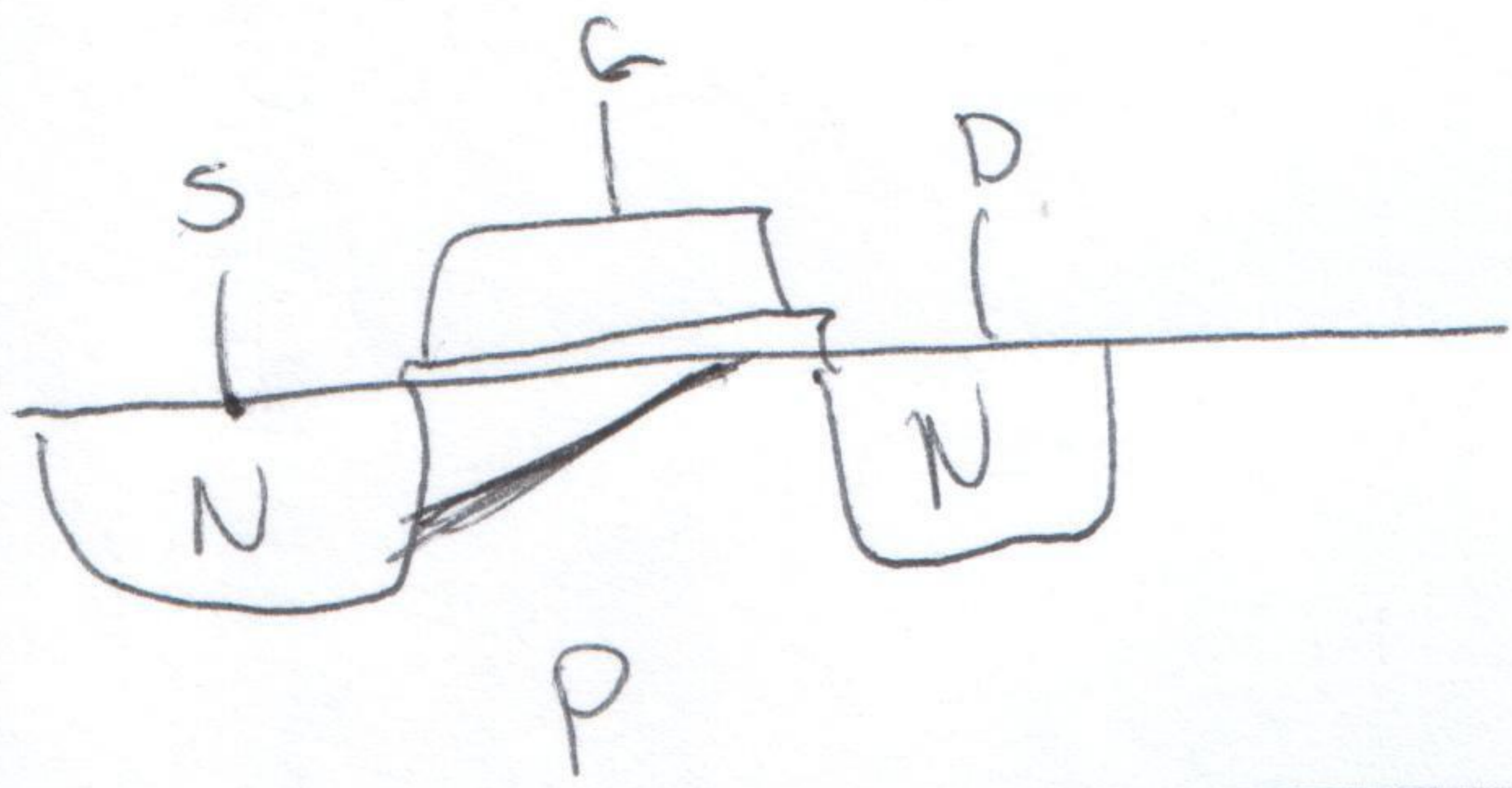


$$V_{CC} - V_L = \left[ k_n' \frac{W}{L} \cdot R \right] \left( V_{GS} - V_{TN} - \frac{V_L}{2} \right) V_L$$

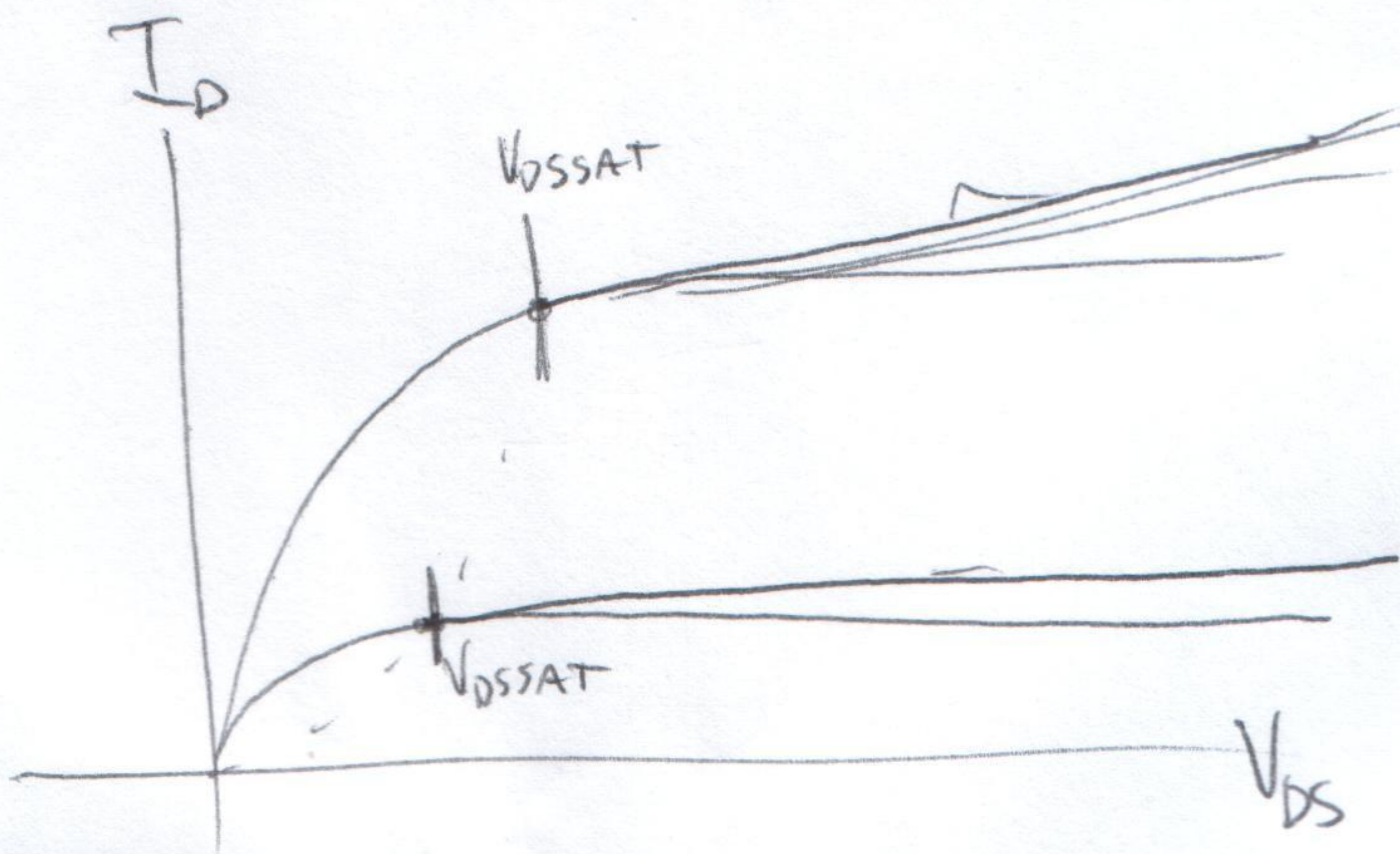
$$\underbrace{k_n' \frac{W}{L} \cdot R}_{\text{resistive}} = \frac{V_{CC} - V_L}{\underbrace{\left( V_{GS} - V_{TN} - \frac{V_L}{2} \right) V_L}_{\substack{\text{Dimensionless - isL} \\ \text{w/ of voltages}}}}$$



# Impact of Channel Width Modulation



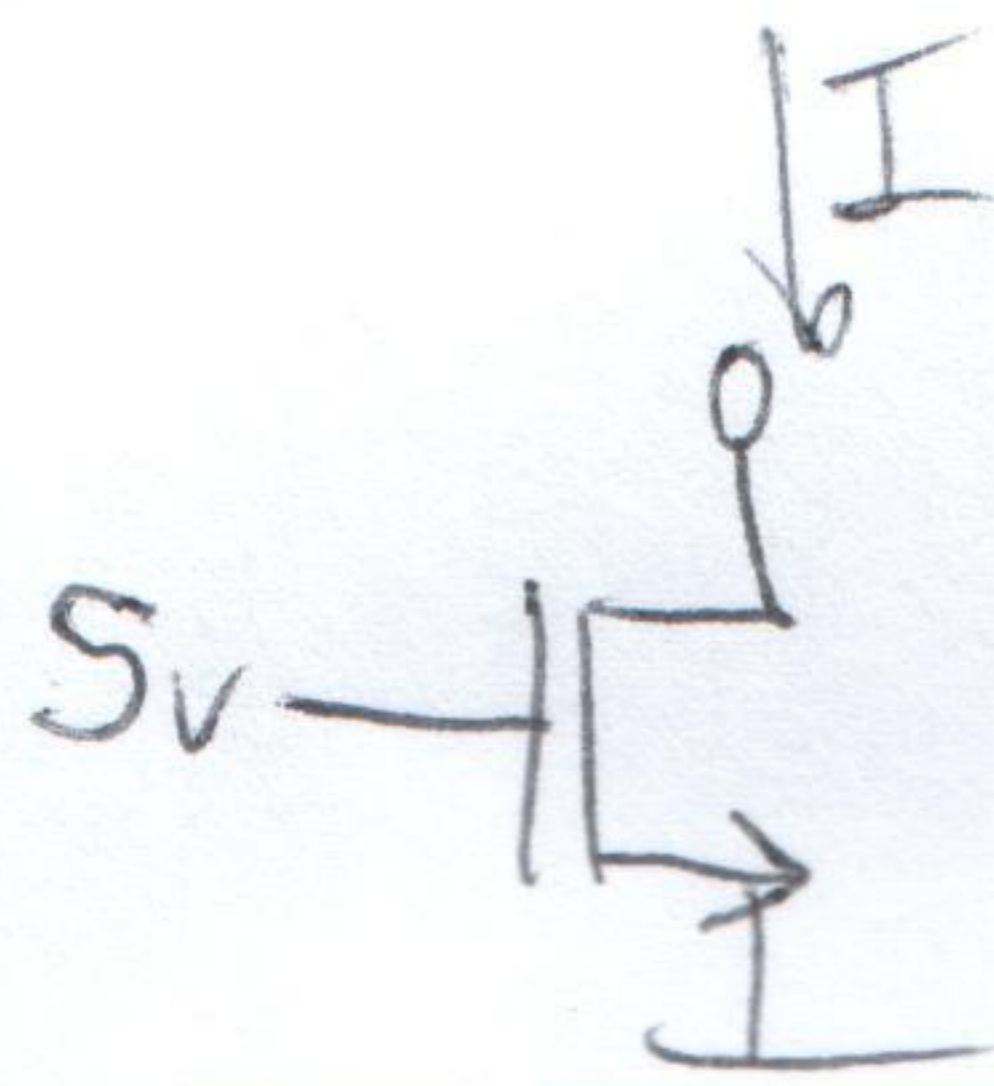
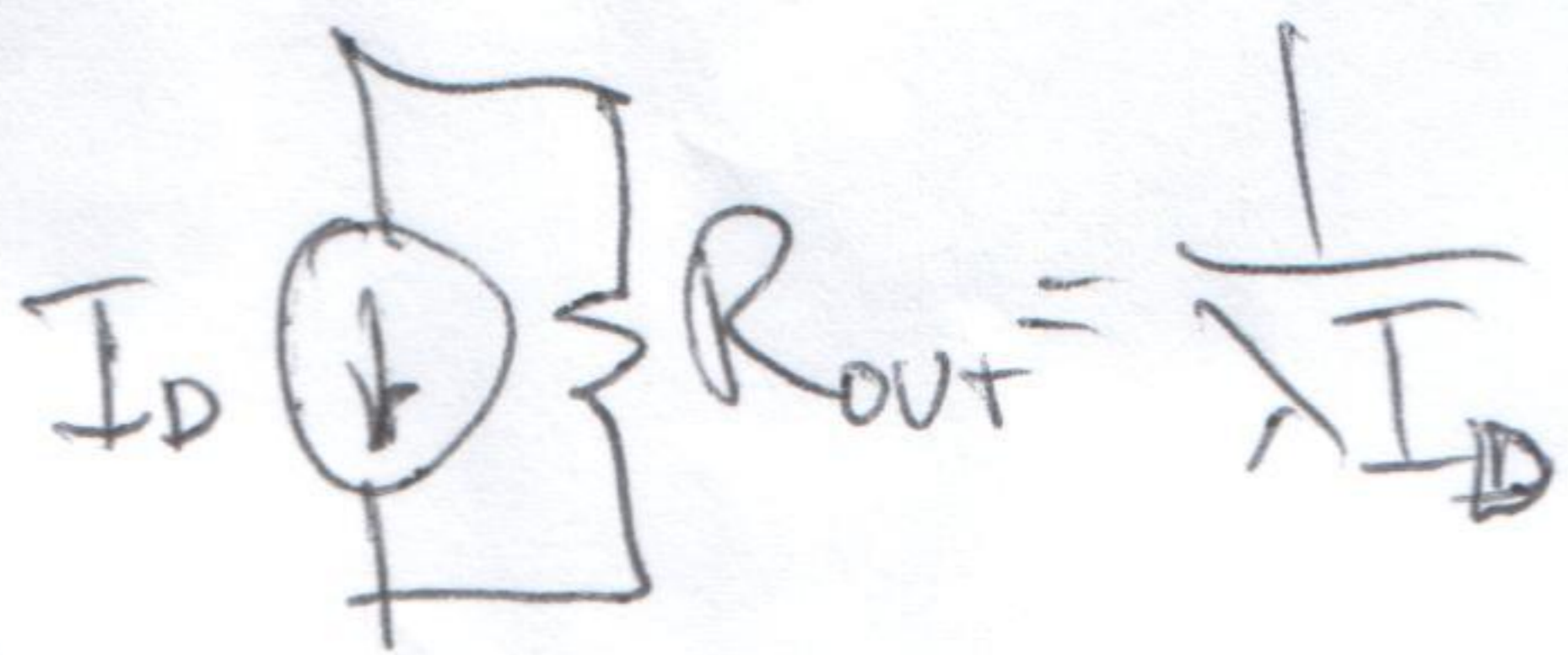
$V_{DS} \rightarrow$  bigger  
 Conductivity in channel  $\rightarrow$  higher



$$I_D' = I_D (1 + \lambda (V_{DS} - V_{DSSAT}))$$

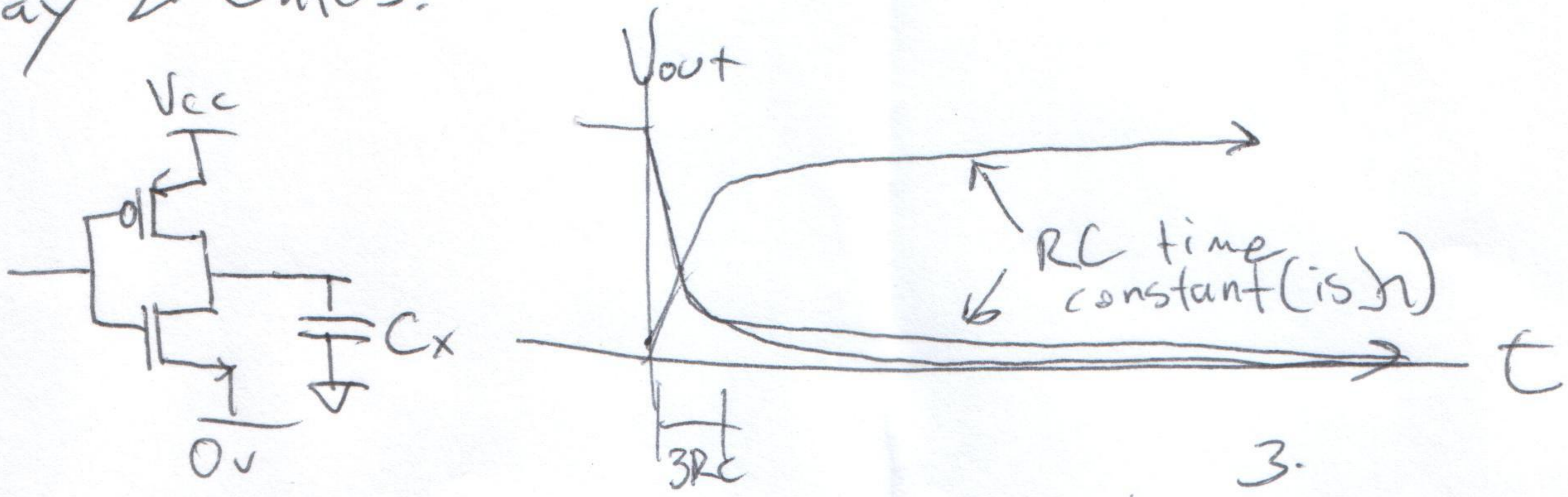
$$V_{DSSAT} = V_{GS} - V_{TH}$$

from inequality

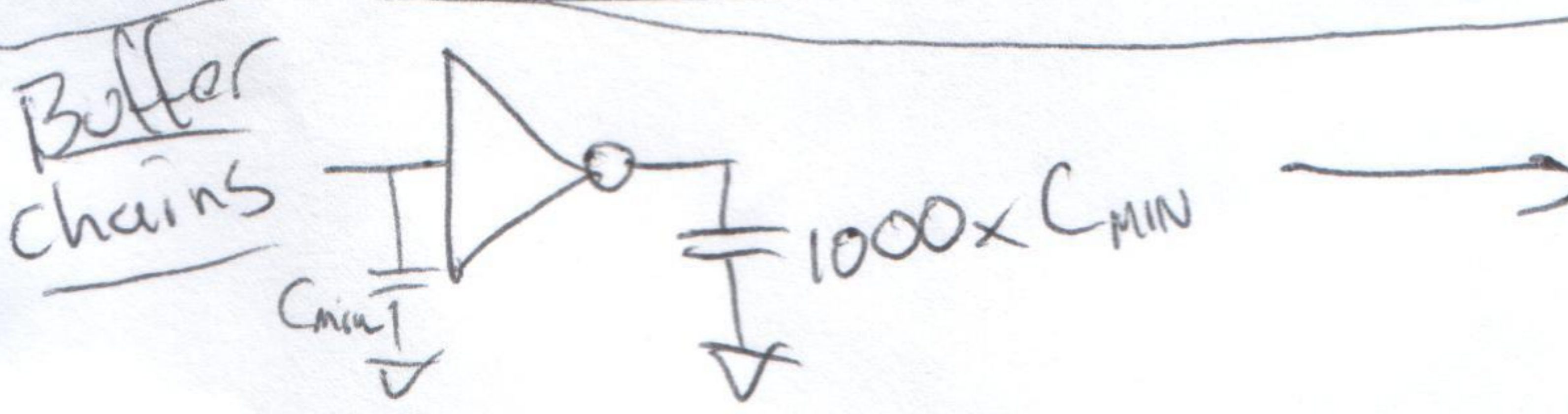
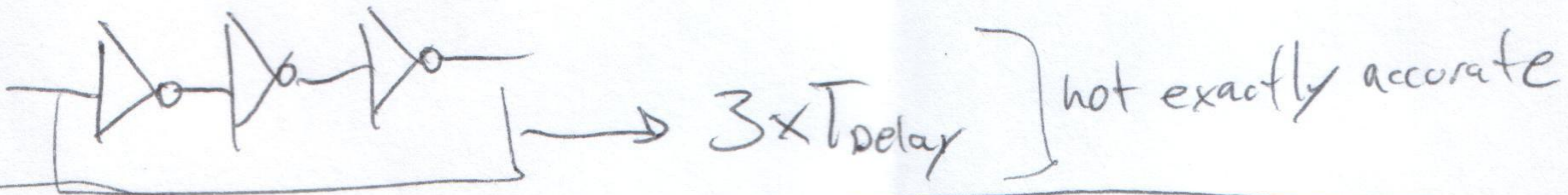


Current source  
 no longer ideal.

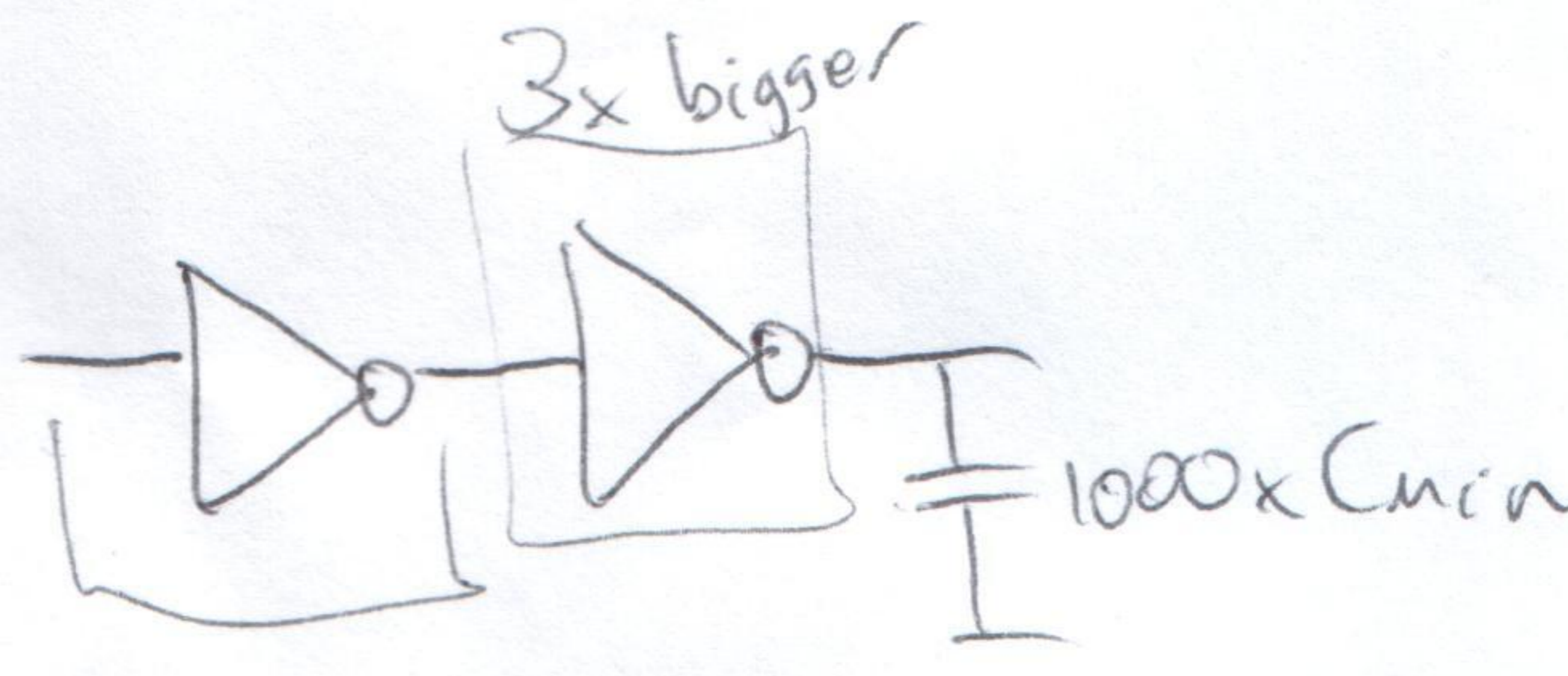
# Delay & Gates:



$T_{delay} =$  lots of things, let's go with  $4 \cdot RC$  3.

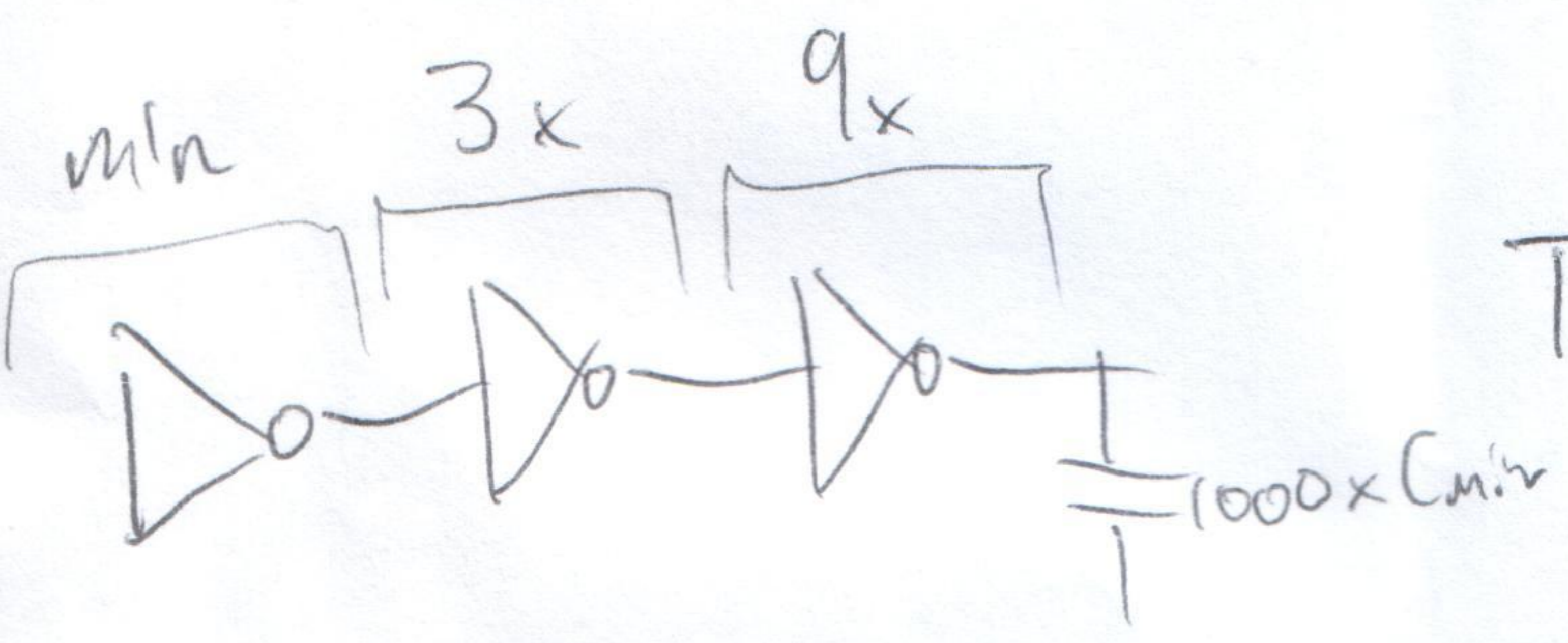


$$T_{delay}' = 1000 \times T_{delay}$$



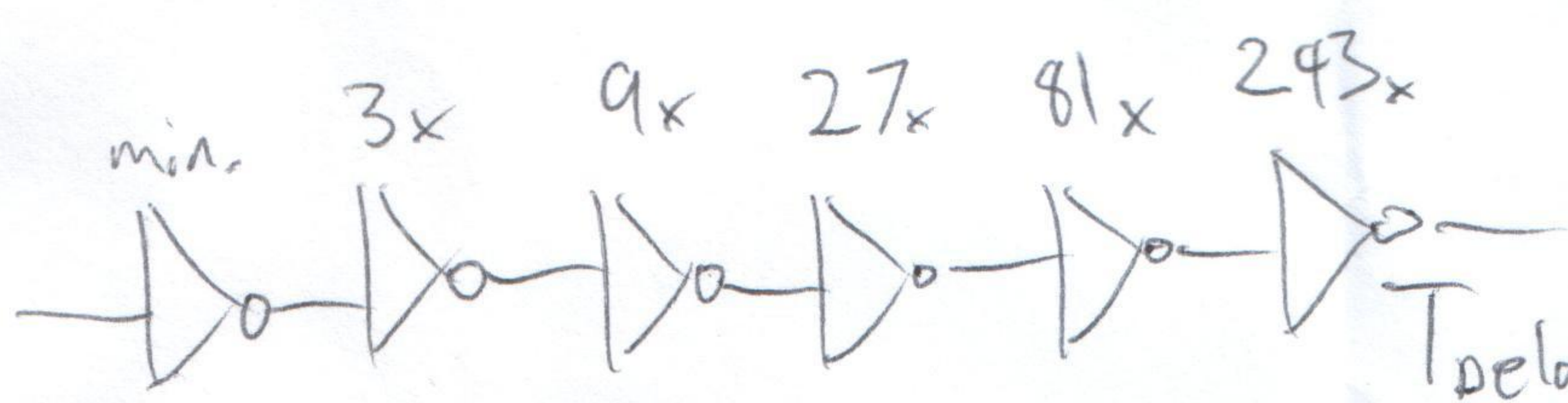
$$T_{delay}' = 3 \times T_{delay} + \frac{1000}{3} T_{delay}$$

$$= 336 \times T_{delay}$$



$$T_{delay}' = 3 \times T_{delay} + 3 \times T_{delay} + \frac{1000}{9} T_{delay}$$

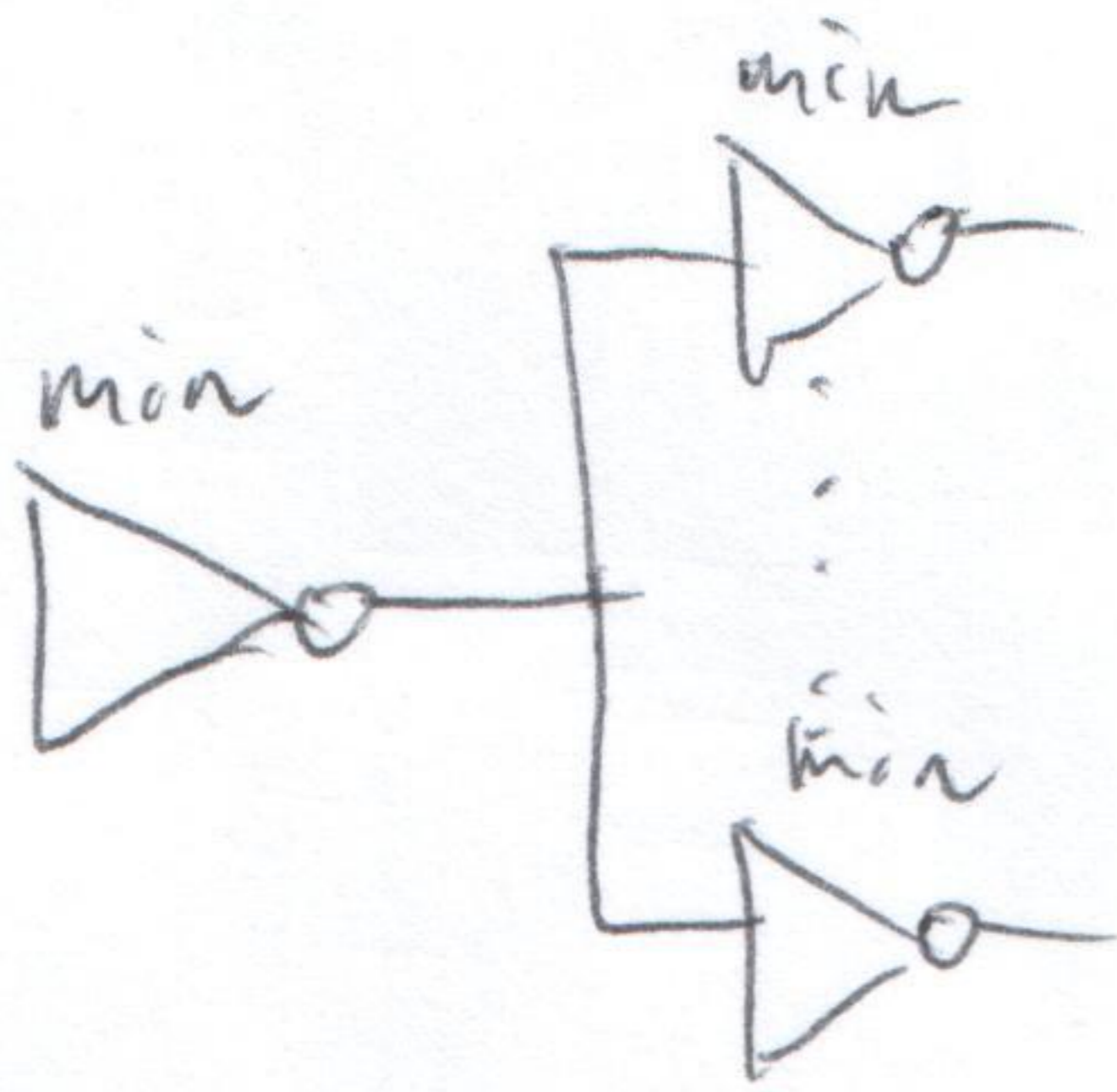
$$= 117 \times T_{delay}$$



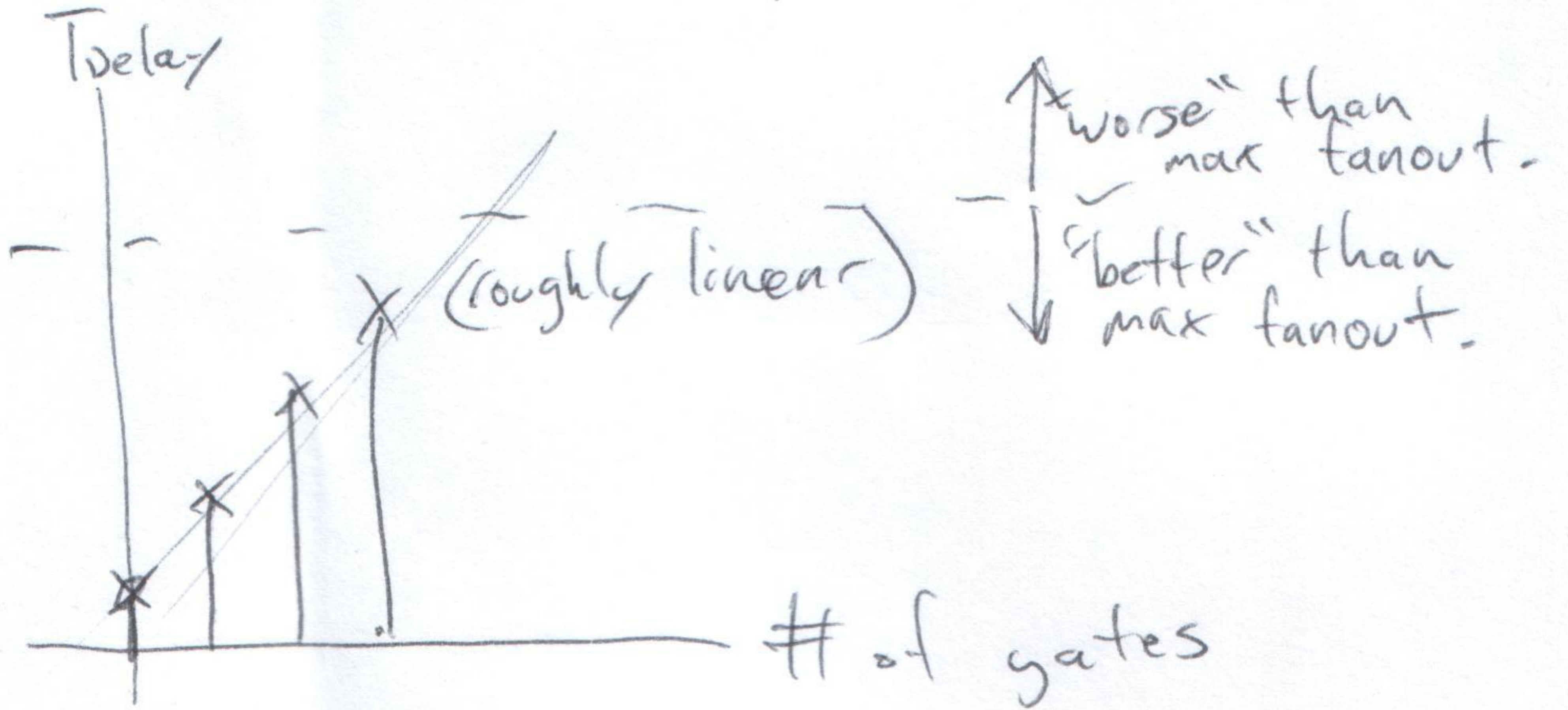
$$T_{delay}' = 15 \times T_{delay} + \frac{1000}{243}$$

$$\approx 20 \times T_{delay}$$

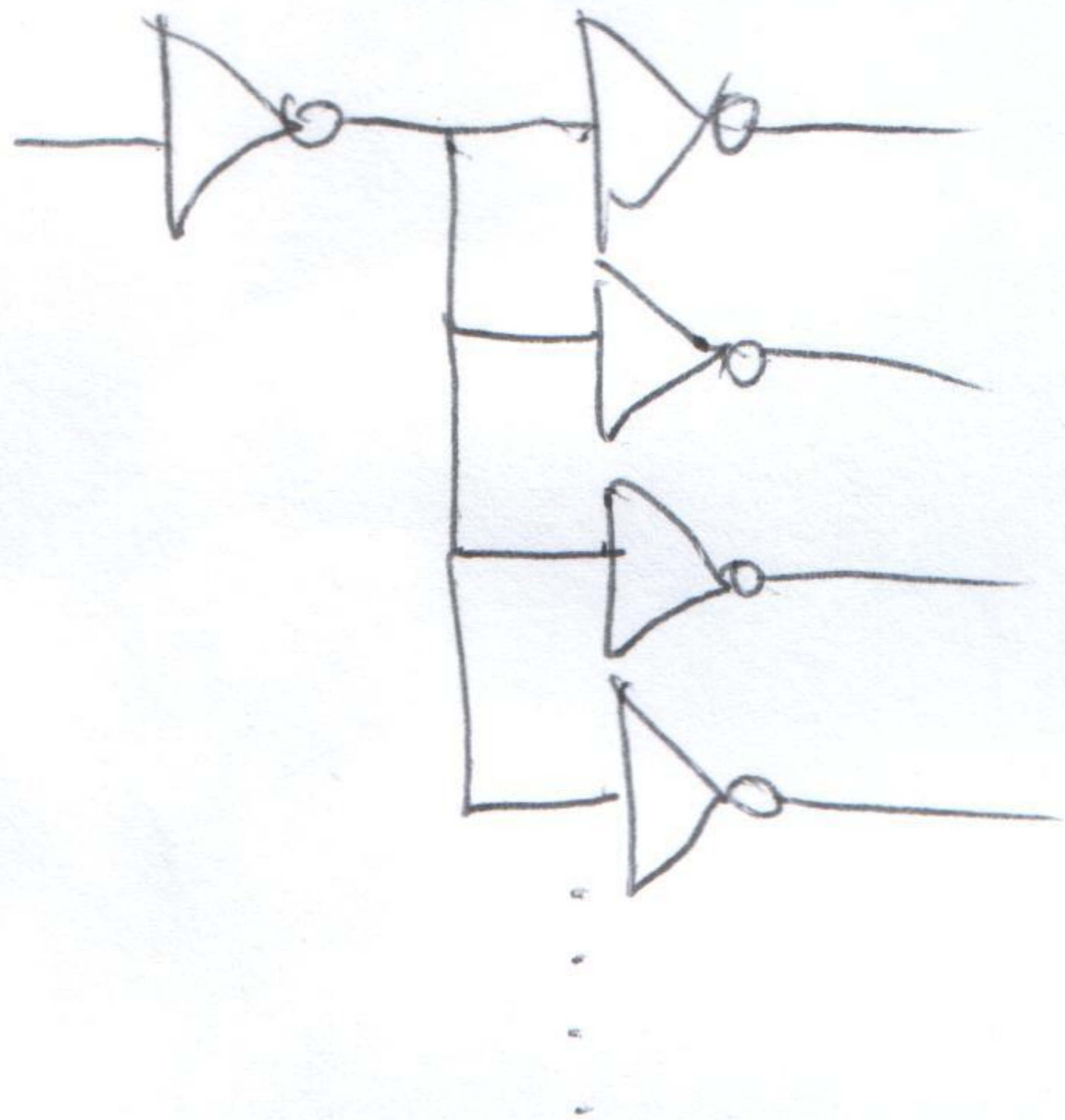
# Related: Fanout



Gate fanout is not a precise science.



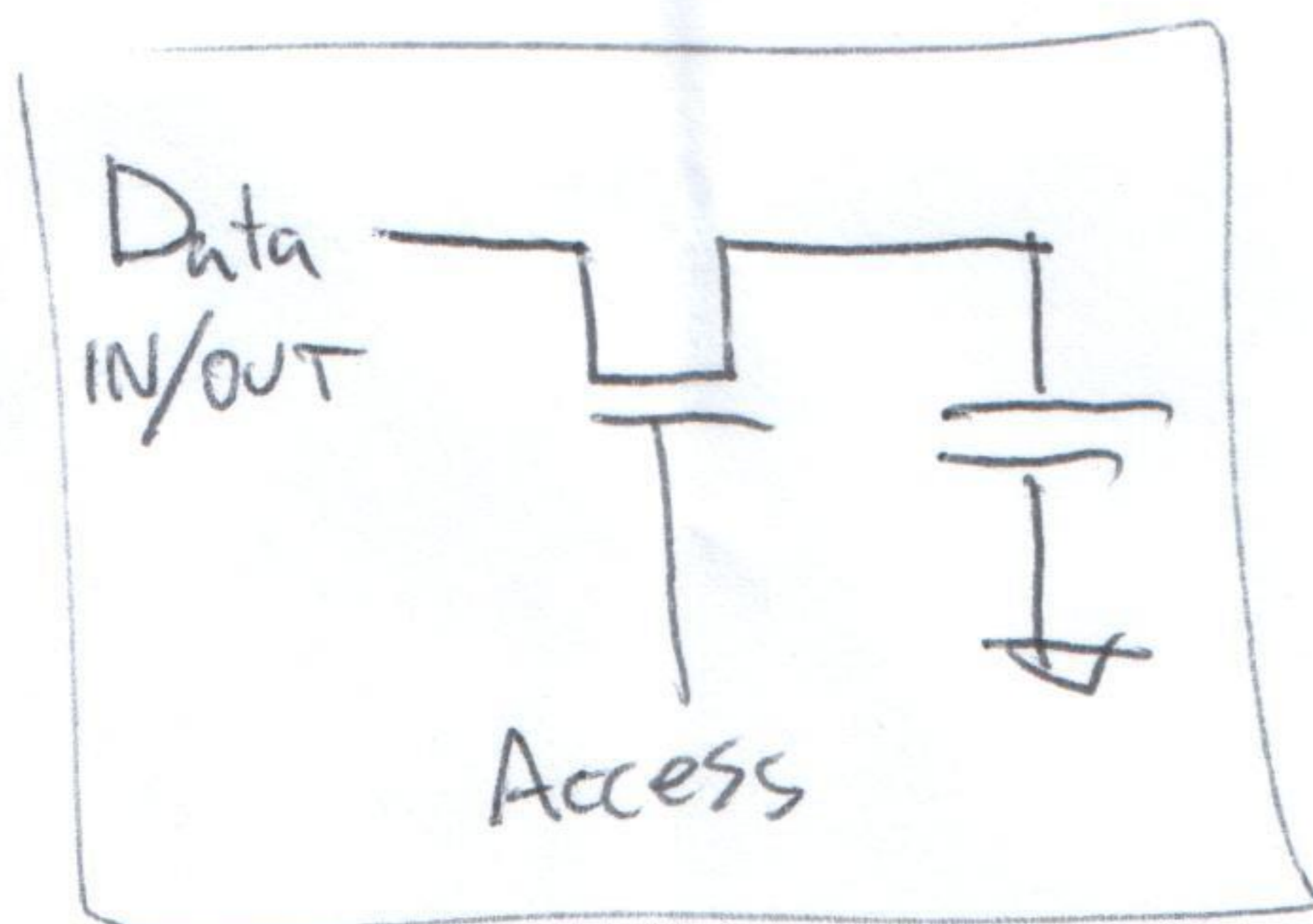
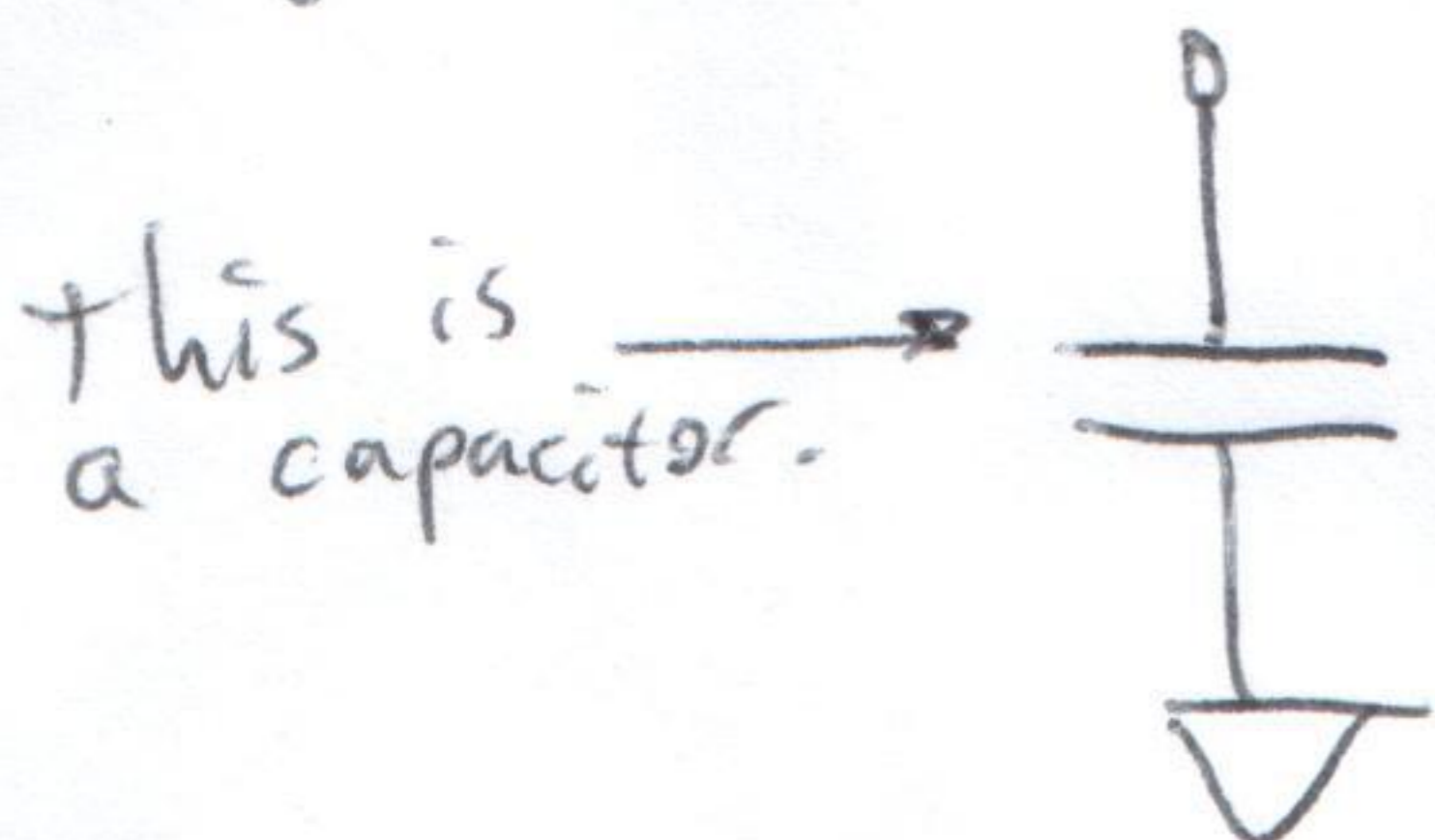
This distinction has more to do with what we just talked about:



at what point does it start to make more sense to buffer a logic stage?

"Fun" fact: it's possible to tune every gate in a system to have close to the same RC constant by sizing everything and/or buffering. Some chips might even do this for super-high-speed logic.

# Storage.



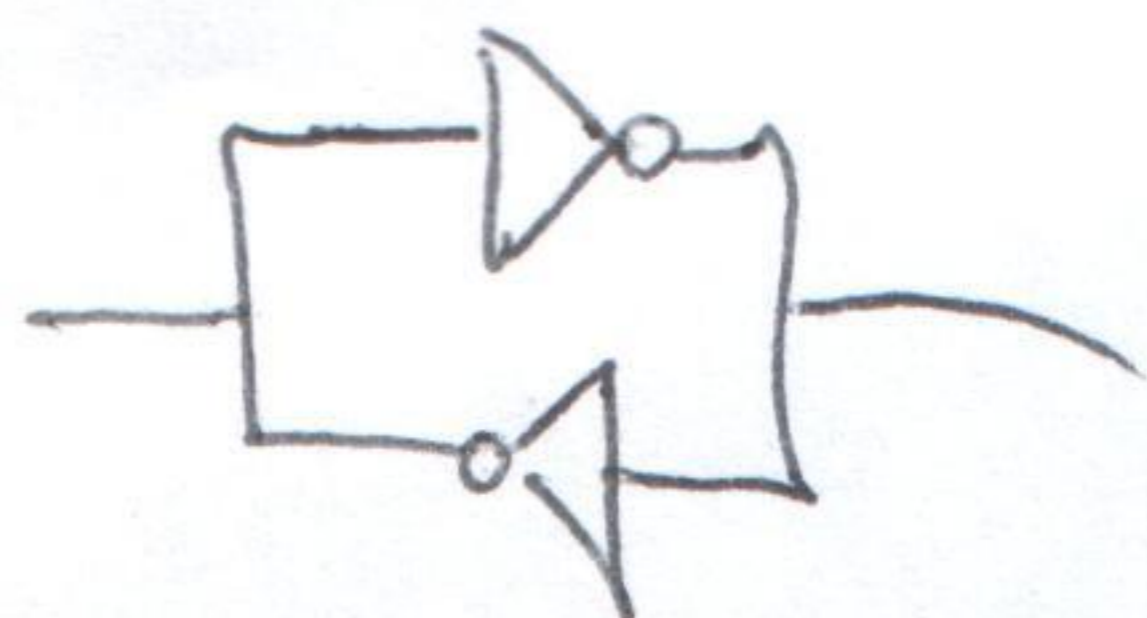
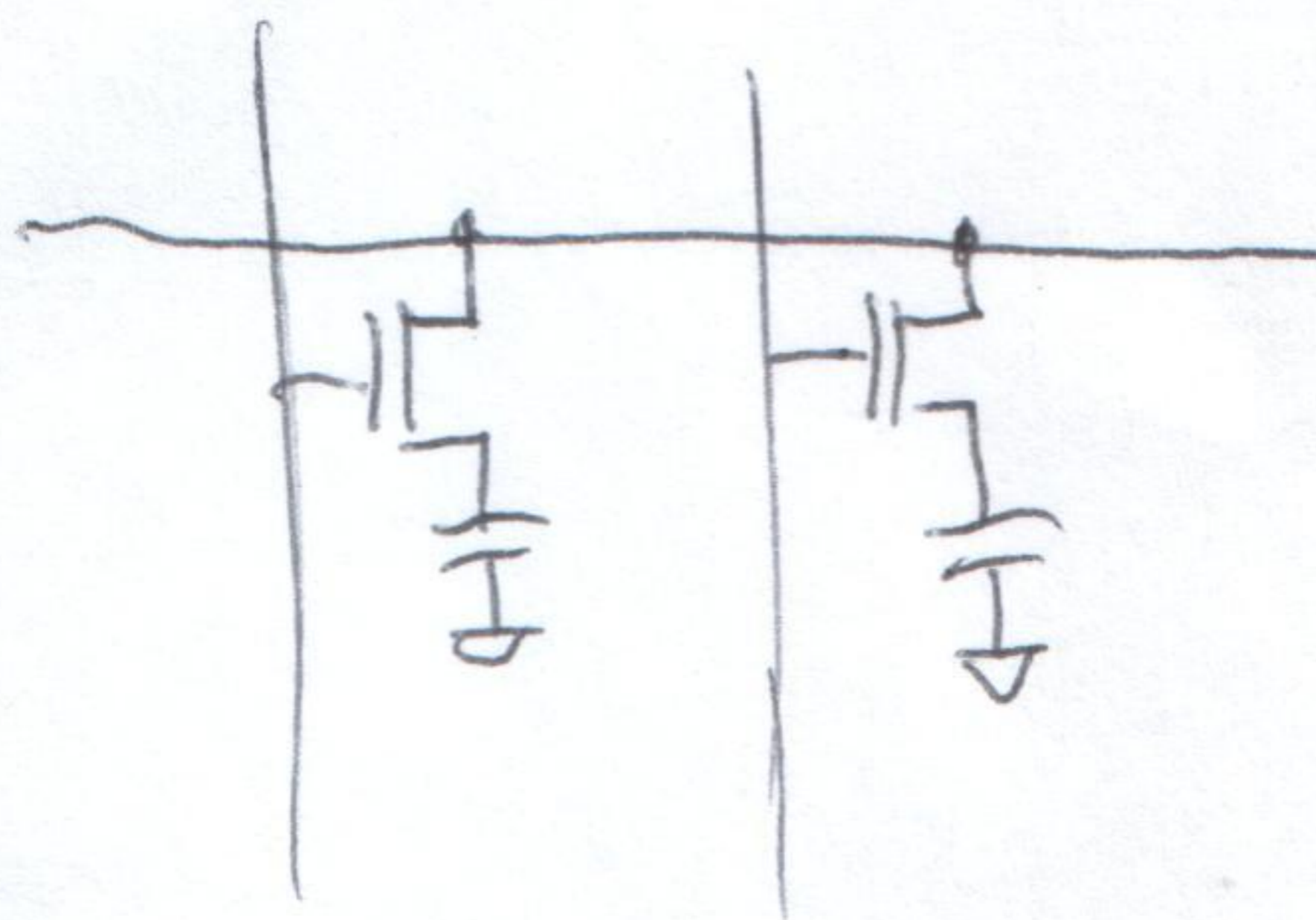
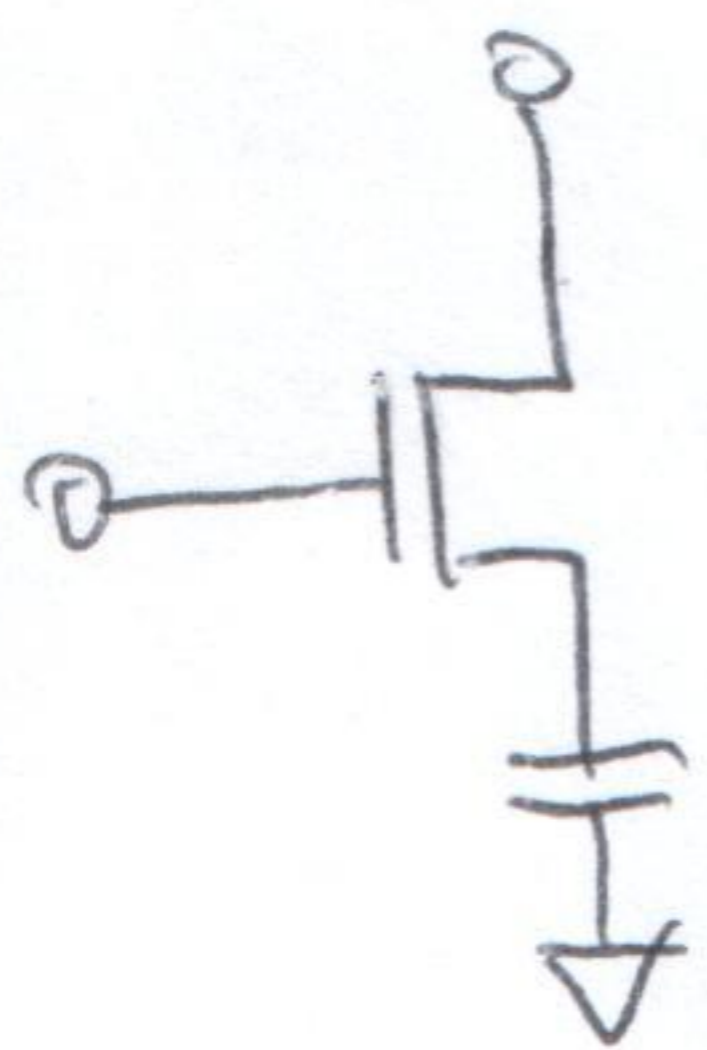
this is a DRAM cell

# Body Effect

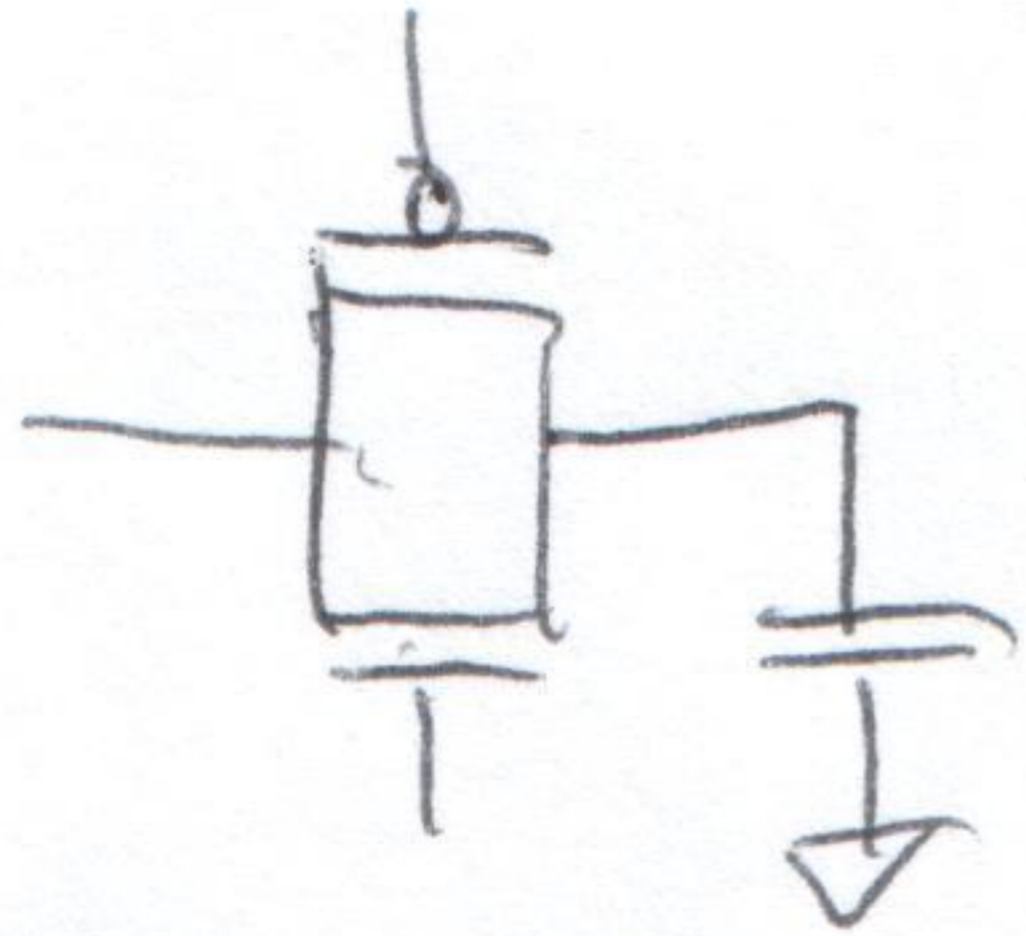
$$V_{TB} = V_{T0} + \gamma \left( \sqrt{V_{SB} + 2\psi_B} - \sqrt{2\psi_B} \right)$$

mod. param.       $V_{BUILT-IN}$

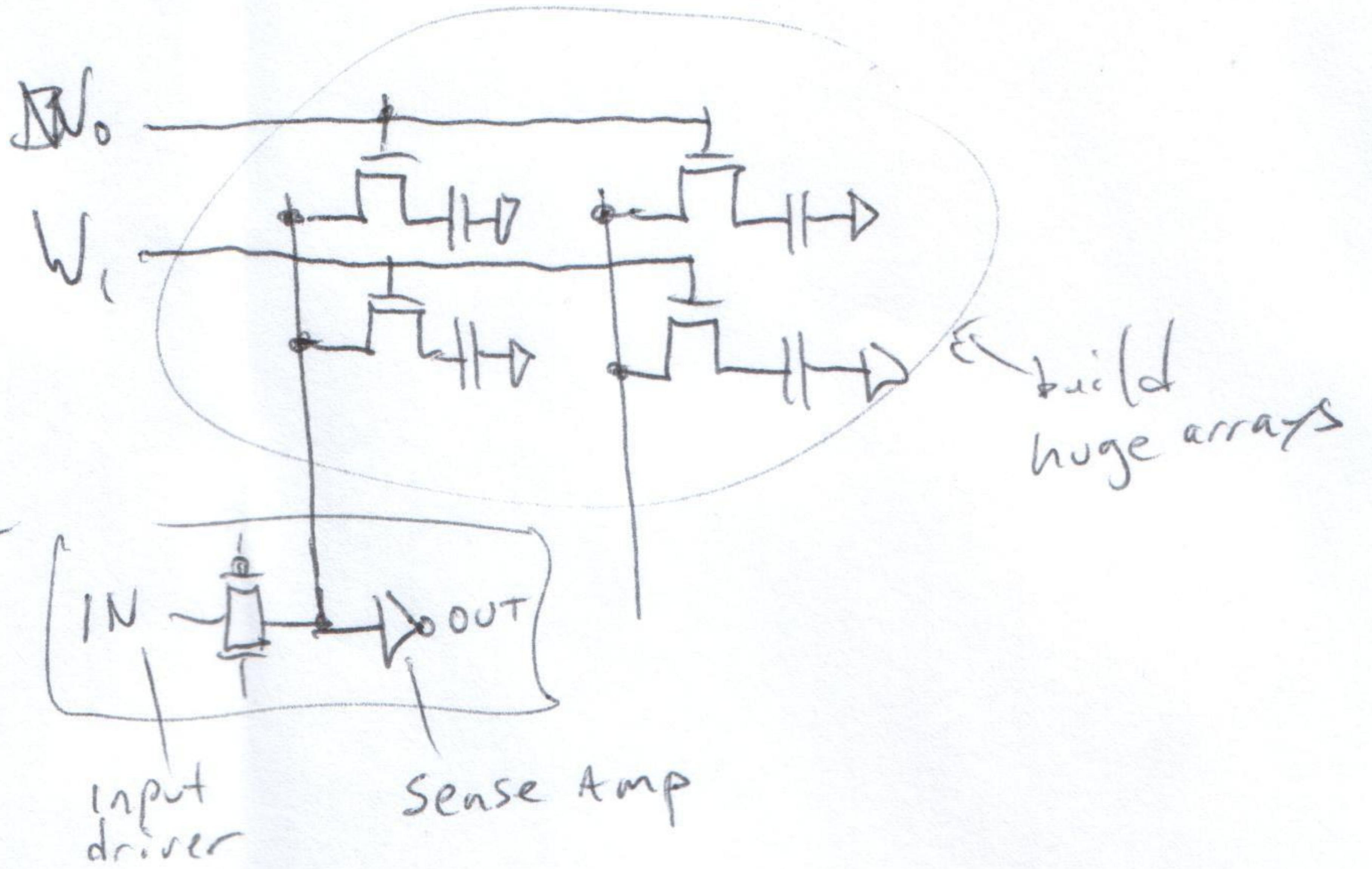
wow, that's awful...







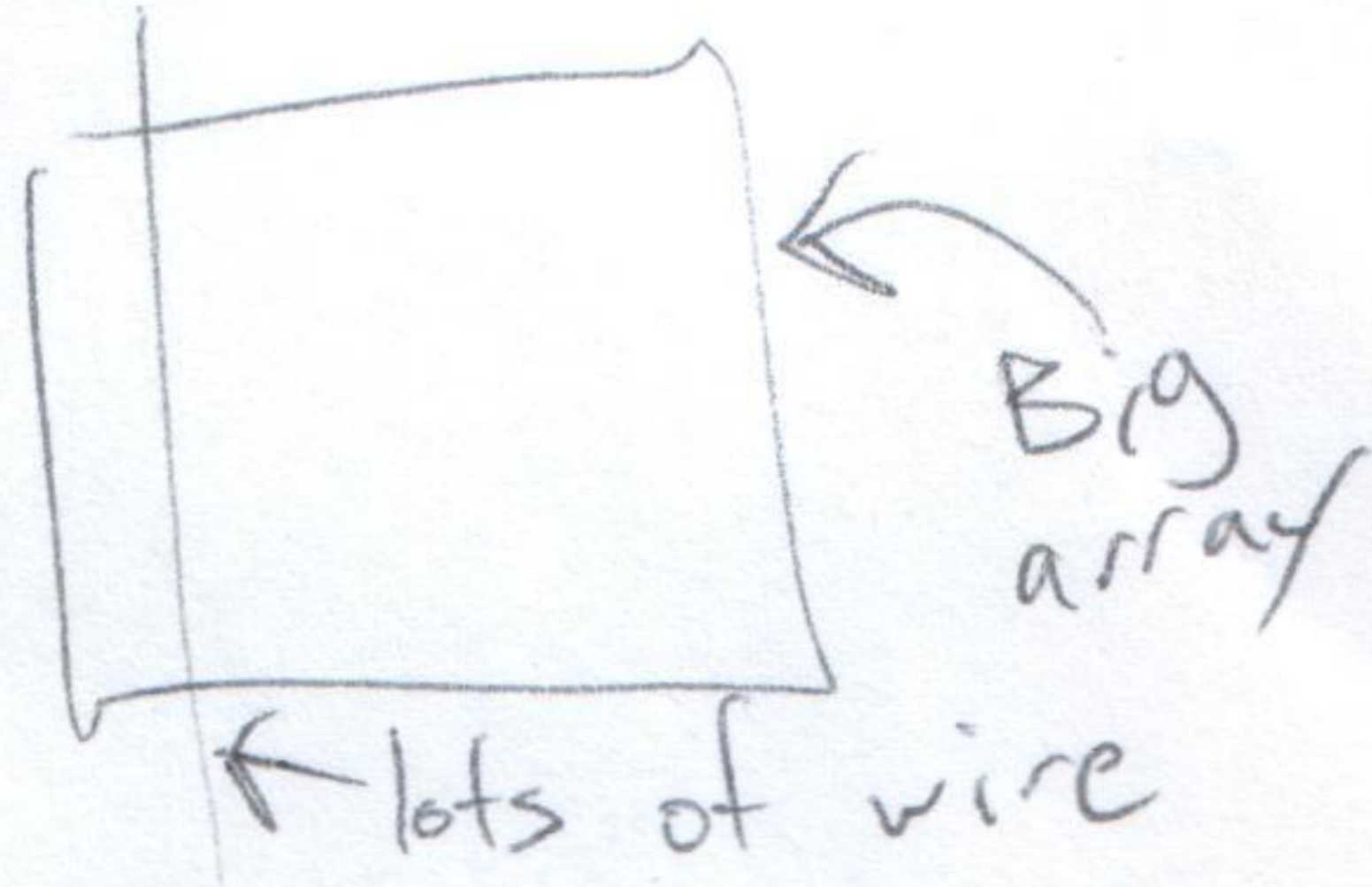
You -could- do this, but that would require 2 transistors.



Problem:



Not a giant capacitor.



Reading costs charge, so reload after you read!

Also:



NOT a perfect open circuit.

Current will leak out quickly!

Solution?!  
Literally just read all the cells and reload every so often

(I am not kidding.)